Eur. Phys. J. D **22**, 183–188 (2003)
DOI: 10.1140/epjd/e2002-00239-3

THE EUROPEAN PHYSICAL JOURNAL D

Instabilities of an electron cloud in a Penning trap

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Received 5 August 2002 / Received in final form 14 October 2002
Published online 17 December 2002 –

Abstract. We have measured the storage instabilities of electrons in a Penning trap at low magnetic fields. These measurements are carried out as a function of the trapping voltage, for different magnetic fields. It is seen that these instabilities occur at the same positions when the trapping voltage is expressed as a percentage of the maximum voltage, given by the stability limit. The characteristic frequencies at which these instabilities occur, obey a relation that is given by $n_z\omega_z + n_+\omega_+ + n_-\omega_- = 0$, where ω_z, ω_+ and ω_- are the axial, perturbed cyclotron and the magnetron frequencies of the trapped electrons respectively, and the n's are integers. The reason for these instabilities are attributed to higher order static perturbations in the trapping potential.

PACS. 52.27.Aj Single-component, electron-positive-ion plasma – 82.80.Qx Ion cyclotron resonance mass spectrometry

1 Introduction

Penning traps [1] have been used in recent years for a variety of experiments: measurement of electronic or nuclear g factors, [2–5], high precision mass measurements [6–9], or plasma studies [10,11]. Three dimensional confinement of charged particles is obtained by a static electric field applied between electrodes and superposition of a magnetic field. Storage times of many hours or even days can be obtained under well defined conditions and the stored particles are then subject to investigations.

In many of these experiments the trap serves merely as a container which keeps the ions in place. Details of the ion motion inside the trap are of interest only when the motional frequencies are to be measured. This is the case in mass spectrometry where the mass dependence of the motional frequencies are used for comparison of the masses of different ions. High precision measurements require the trap's potential to be harmonic since in this case the oscillation frequency does not depend on the ion's position and energy. Deviations from the harmonic potential result in shifts of the motional frequencies which would limit the precision.

The standard Penning trap design (Fig. 1) has two endcap electrodes and a ring electrode rotationally symmetric around an axis (z-axis). The superimposed magnetic field is directed along the z-axis. Harmonicity of the potential is obtained when the surfaces of the trap's electrodes follow hyperboloids of revolution. The potential inside this arrangement is a quadrupole potential,

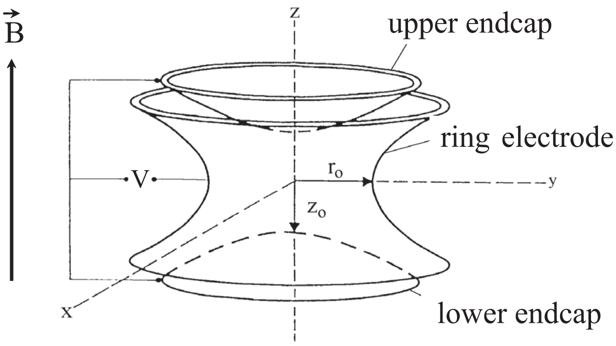

Fig. 1. Penning trap electrodes.

$$\Phi = \frac{V}{2d^2}(\rho^2 - 2z^2) \tag{1}$$

where d is the characteristic dimension of the trap, $d^2 = {r_0}^2 + 2{z_0}^2$, r_0 is the ring radius and $2z_0$ the distance between the trap electrodes. $\rho = [x^2 + y^2]^{1/2}$ is the radial coordinate. The motion of an ion inside this potential can be described by three harmonic oscillations: an oscillation along the z-axis (frequency ω_z), a perturbed cyclotron oscillation perpendicular to the magnetic field direction with frequency ω_+, and a slow magnetron drift of the cyclotron orbits around the trap center with frequency ω_-. The values of these frequencies depend on the trapping

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parameters:

$$\omega_z = \sqrt{\frac{qV}{md^2}} \tag{2}$$

$$\omega_+ = \frac{\omega_c}{2} + \sqrt{\frac{{\omega_c}^2}{4} - \frac{{\omega_z}^2}{2}} \tag{3}$$

$$\omega_- = \frac{\omega_c}{2} - \sqrt{\frac{{\omega_c}^2}{4} - \frac{{\omega_z}^2}{2}} \tag{4}$$

$\omega_c = (q/m)B$ is the cyclotron frequency of the free particle with charge q and mass m.

In a real trap the potential can deviate from the ideal quadrupole geometry by imperfect shapes of the electrode surfaces, by a tilt angle between the trap's axis and the direction of the magnetic field, and by the Coulomb potential in case of a stored particle cloud, which contributes higher order terms to the trapping potential. A consequence of existing imperfections is that the motional eigenfrequencies are shifted. These shifts scale with the size of the imperfections and the ion energy. They are of great importance in precision mass spectrometry and a great amount of effort has been devoted to the calculation of these shifts theoretically [12–15] and to minimize the shifts experimentally by correction electrodes [16–18]. Another consequence of these imperfections is that the motion of charged particles may become unstable at certain operating parameters of the trap. In case of a Paul trap where a time dependent electric field serves to create a time-averaged three-dimensional potential minimum for particle trapping, such ion loss has been observed and investigated in some detail [19,20]. In a Penning trap instabilities have been observed for the first time by Schweikhard and coworkers [21] at some operating points with heavy cluster ions as trapped particles. Later this has been confirmed and extended in laser spectroscopic measurements on Ba ions [22]. Such storage instabilities are also of significance in accelerators and storage rings [23] and considerable effort is devoted to minimizing them.

In this work, we investigate the appearance of instabilities in a Penning trap when a cloud of charged particles is stored. We use electrons for our investigations since they are easier to produce than atomic ions. Also electron detection can be performed by simple electronic means without use of laser excitation and fluorescence detection. Moreover storage of electrons is possible at relatively low magnetic fields. The results, however, are independent of the nature of the particles and are valid for different ions.

2 Apparatus

Our Penning trap has hyperboloid-shaped electrodes. The ring radius is 2 cm. It was previously used in laser spectroscopic experiments and therefore required holes of a few mm diameter in the ring electrode and slits in one of the endcaps. Moreover, the other endcap was made of a metallic mesh to allow the transmission of fluorescence from a trapped ion cloud. These modifications as well as the truncation of the electrodes introduce deviations of the potential from the ideal quadrupolar type. The magnetic field in the z-direction, produced by two coils in approximate Helmholtz configuration, can be varied between 0 and 10 mT. The direction of the magnetic field lines inside the trap may have an angle with respect to the trap axis. Electrons were injected into the trap from a hot tungsten wire just below one endcap by a negative pulse of 100 V amplitude and typically 10 ms length, applied to the filament. This limits the possible angle between the magnetic field and the trap axis by the 0.5 mm diameter of the entrance and exit holes in the endcap electrodes to 0.015 rad. While the endcap electrodes were held at ground potential, a constant positive storage voltage was applied to the ring for a time which could be varied between 10 ms and virtually infinity, before being ramped down to negative values.

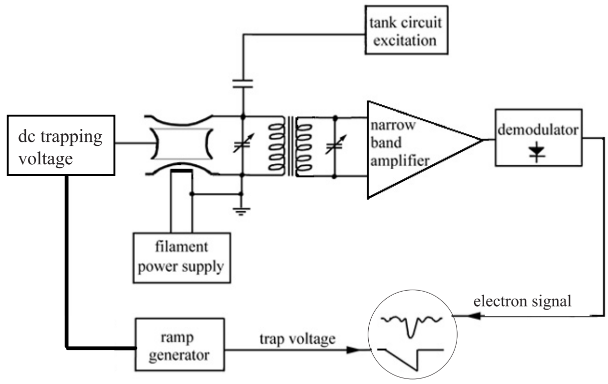

Fig. 2. Setup for electron storage and detection.

Detection of trapped electrons was facilitated by connecting a tank circuit consisting of an inductance and a capacitance in parallel to an endcap electrode (Fig. 2). It was weakly excited at its resonance frequency ω_{LC} ($2\pi \times$ 20 MHz). When the trapping voltage V is ramped down the electron's axial frequency ω_z changes according to equation (2). For a particular value of the voltage it coincides with ω_{LC}, leading to an energy transfer from the circuit to the electrons. The corresponding damping of the circuit was detected as a drop in the measured voltage across the endcap (Fig. 3). After rectification we obtained a signal whose amplitude is proportional to the number of trapped electrons, which was then digitized for further handling. The ramp voltage was lowered until it changed sign to ensure that all electrons leave the trap after being detected. This was followed by a new cycle of electron creation, storage and detection. From the size and shape of the detection signal the number of trapped electrons can be estimated knowing the gain of the detection electronics [24]. A typical number is 10^5. This order of magnitude is confirmed by the observed space charge shift of the axial oscillation frequency from which a number can be deduced using a simple model for the electron density distribution [25].

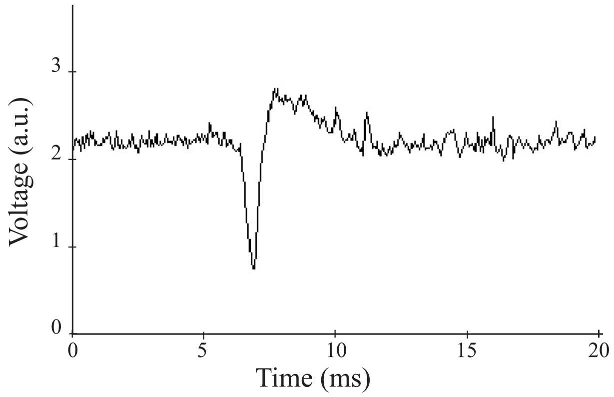

Fig. 3. Damping signal from a stored electron cloud.

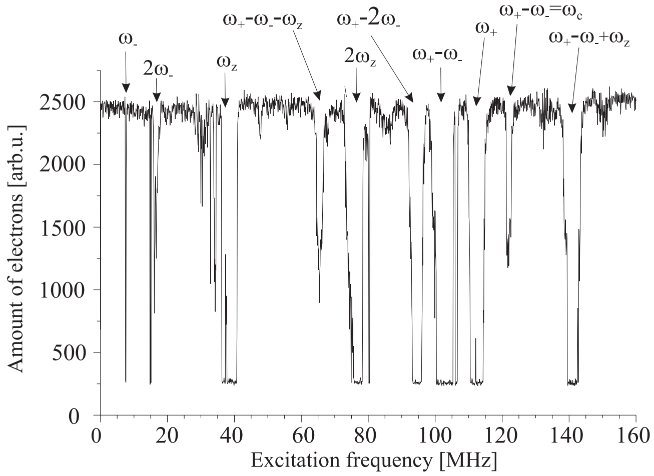

Fig. 4. Excitation spectrum of motional frequencies of the electron cloud.

3 Measurements and results

We measured the motional resonances of the electrons by applying an additional r.f-field capacitively coupled to the ring electrode using it as antenna, and sweeping the frequency of this field. The electrons become excited whenever the frequency coincides with one of the motional modes. Some of the electrons leave the trap and the motional resonances appear as minima in the detection signal. Figure 4 shows such a spectrum which contains the fundamental oscillation frequencies ω_- (magnetron), ω_z (axial), and ω_+ (perturbed cyclotron) as well as some linear combinations or multiples of these frequencies. The number of observed combinations depends on the amplitude of the applied r.f-field. Identification of the pure cyclotron frequency, ω_c, enables us to determine the magnetic field accurately, in the trapping region.

We also measured the number of trapped electrons (in relative units) for a given trapping potential and a fixed magnetic field and averaged over a few detection signals to reduce the statistical scatter of the signal height. The amplitude of the signal was stable within a few percent. The trapping voltage was incremented by a small amount and a new value of the signal height recorded. The lowest trapping voltage in our experiment was 10 V, and was the voltage at which the electron's axial frequency ω_z equals the resonance frequency of the tank circuit. The maximum trapping voltage is decided by the maximum allowed value for stable confinement for a given magnetic field. This is derived from the condition that the term under the root appearing in equations (3, 4) must be positive:

$$\omega_c{}^2 \geq 2\omega_z{}^2 \tag{5}$$

from which it follows that

$$V_{\max} = \frac{q}{2m} d^2 B^2. \tag{6}$$

Data were taken at different values of the magnetic field and for different storage times, ranging from 10 ms to several seconds. When we plot the measured electron number for a fixed magnetic field and a fixed storage time as function of the trapping voltage we observed a reduction of the electron number at some discrete points for small values of V. They became more pronounced at higher voltages until finally no electrons could be detected at all. Figure 5 shows an example for $B = 3.2$ mT and storage times up to 3 600 ms between electron creation and detection. Similar studies were conducted for different values of the B-field. Generally, no electrons could be observed when the trapping voltage exceeded about half the value calculated from the stability criterion (Eq. (6)). More and broader minima in the detection signal appeared for longer storage times. At storage times of a few seconds they were extremely broad indicating that stable storage of electrons was impossible at these storage times.

4 Discussion

The experimental results can be displayed in a general way when we normalize the trapping voltage to the calculated voltage limit at each magnetic field as in Figure 6. Then it is seen that the instabilities occur always at certain fractions of the normalized voltage. These positions are given by a simple relation between the motional frequencies of the electrons:

$$n_z\omega_z + n_+\omega_+ + n_-\omega_- = 0 \tag{7}$$

where n_z, n_+, n_- are integers. Figure 6 indicates the positions of these instabilities for a few sets of integers. In order to predict the sets of integers (n_z, n_+, n_-) where storage instabilities occur, we follow an analysis as suggested by Kretzschmar [26]. By rewriting (7), using (3) and (4), we obtain,

$$(n_+ + n_-)\omega_c + 2n_z\omega_z = (n_- - n_+)\sqrt{\omega_c{}^2 - 2\omega_z{}^2}. \tag{8}$$

Squaring this on both sides, leads to a linear relation between ω_z and ω_c:

$$\omega_z = K\omega_c \tag{9}$$

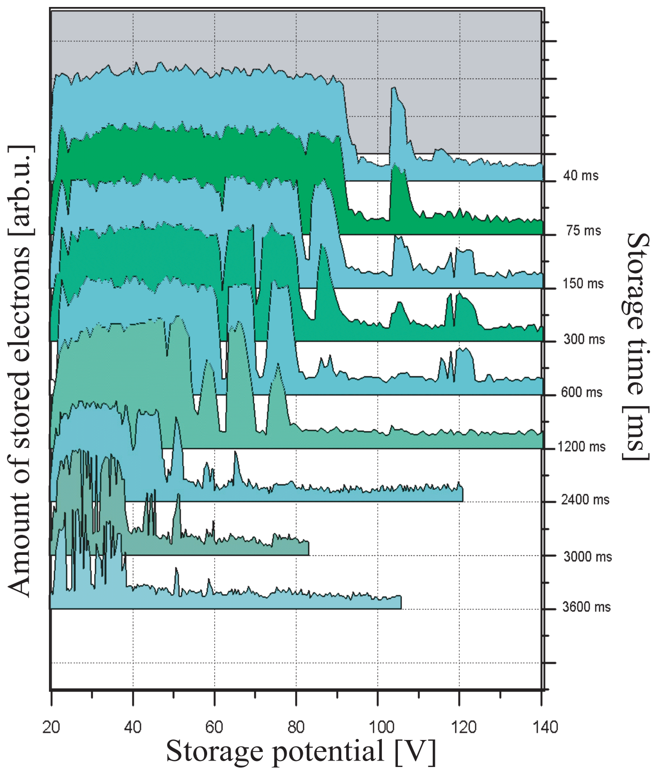

Fig. 5. Detected electron number (in arbitrary units) as a function of the storage potential. Data were taken at a fixed magnetic field (3.2 mT) and for different storage times.

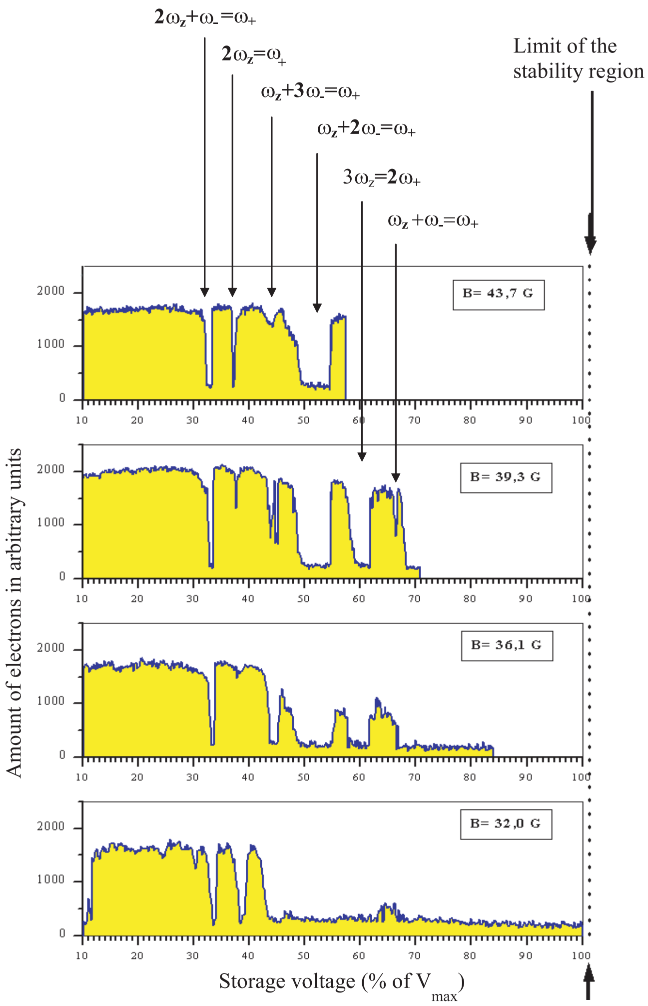

Fig. 6. Stored electron number (in arbitrary units) for different magnetic fields as a function of the trapping voltage. The voltage is normalized to the stability limit given by equation (6). The storage time is fixed to 600 ms.

where K is a proportionality constant related to the integers and is given by

$$K = \frac{B \pm \sqrt{B^2 - AC}}{A} \tag{10}$$

$$A = 2(n_+ - n_-)^2 + 4{n_z}^2 \tag{11}$$

$$B = -2(n_+ + n_-)n_z \tag{12}$$

$$C = 4n_+n_-. \tag{13}$$

The upper limit is given by $K_{\max} = 1/\sqrt{2}$ and represents the stability limit. Figure 7 shows plots for representative values of K obtained from the integer triplets we have recorded in our measurements. Motional instabilities thus may occur, at frequencies that satisfy equation (9) for any particular value of K. The predictions and the experimentally observed instabilities agree on the 1% level which is the uncertainty of the measured voltages at which they occur.

The appearance of instabilities is explained by considering the expansion of the trapping potential in a series of spherical harmonics $P_n(\cos\theta)$ [12]:

$$\Phi = \sum_{n=0}^{\infty} c_n \left(\frac{r}{r_0}\right)^n P_n(\cos\theta) \tag{14}$$

$n = 2$ gives the ideal quadrupole potential and non-zero coefficients c_n indicate the existence of perturbing potentials of order n, which are time-independent and are attributed to imperfections in the shapes of the electrodes, misalignments or space charge effects in the trapping region. In his work, Kretzschmar [13,14] applies perturbation theory formulated in terms of action-angle variables, for determining the shifts in the characteristic frequencies in a perturbed orbit, for perturbations that are time-independent as well as time-dependent. For static perturbations, as in our case, perturbative singularities are shown to occur when the frequencies satisfy equation (7). The details of these calculations may be found in these references and are not elaborated here. In this context it should be mentioned here that the method of Birkhoff normal forms can also be applied [27] in predicting these instabilities.

In addition to theoretically predicting these instabilities, an expansion of the potential in a Fourier series [13] reveals the restriction on the integers in equation (7):

$$|n_+| + |n_-| + |n_z| \le n. \tag{15}$$

From the above equation, it is clear that for a given set of integers that satisfies equation (7), we can assume the

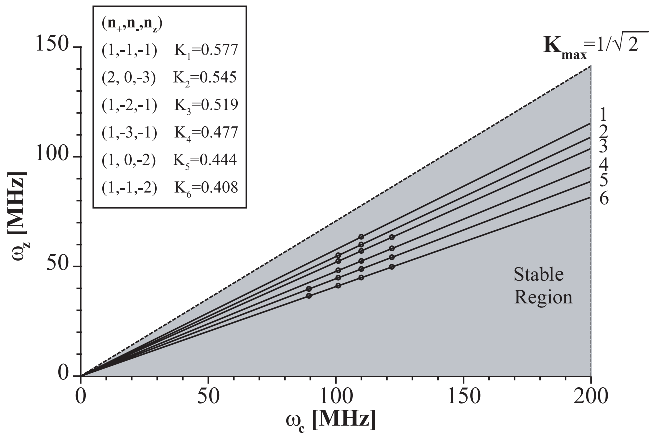

Fig. 7. Lines of instabilities within the stable region of a Penning trap following equation (9). The experimentally observed instabilities are indicated by dots.

likely presence of perturbing potentials at least of order n, that cause the instability. Thus, in our case (Fig. 7), there are perturbations of at least 3rd order (sextuplet) or higher, in the potential. For any given K, in equation (9), there can be more than one integer set [26]. For example, in our case for $K = 0.577$, there are two sets of integers (n_+, n_-, n_z) that can give the same value for K, namely $(1, -1, -1)$ and $(2, -2, -2)$. However we observe the instability relation for $(1, -1, -1)$, and not $(2, -2, -2)$ (Fig. 7). This could be because of the absence of perturbing potentials of order 6 (Eq. (15)) in our trap. On the other hand, for $K = 0.545$, we can also, in principle, observe an instability for integer set $(0, -3, 1)$, which would indicate the likelihood of 4th order perturbations or higher. However, we do not observe any instability that satisfies this particular integer set, despite the presence of other instabilities ($K = 0.519$, $(1, -2, -1)$ or $K = 0.408$, $(1, -1, -2)$) that indicate the presence of 4th order perturbations. The reason for not observing this particular instability could be due to a limitation imposed by the finite storage time of electrons. Nevertheless, it should be borne in mind that equation (15) sets a lower bound on the order of the perturbing potential that may be causing a given instability, but it does not indicate that this order of the perturbation exists definitely.

In the presence of time-dependent perturbations as in Paul traps or the combined trap [28], instabilities have been observed for the characteristic frequencies, that satisfy an equation analogous to equation (7). For the Paul trap [19,20] these occur for the frequencies obeying the following relation:

$$n_r \omega_r + n_z \omega_z + n_\Omega \Omega = 0 \tag{16}$$

where ω_r, ω_z and Ω are the radial, axial and the AC field frequencies respectively and n_r, n_z and n_Ω are integers, Wang *et al.* [29] have solved the nonlinear Mathieu equations to predict equation (16). This instability relation is analogous to equation (7), with the magnetron frequency being replaced with the radial frequency and the perturbed cyclotron frequency, with the AC field frequency. In [13], it is pointed out that the equations of motion for a Penning trap can be transformed to Mathieu equations by rotations to a coordinate frame rotating at $\omega_c/2$ and by the inclusion of a time-dependent driving term. The resultant equations then are solved to predict a relation as given in equation (7). This is also established in an analysis of the combined trap [28] where it is shown that the radial frequency term reduces to $\omega_c/2$ when the magnetic field is finite and the r.f voltage is set to zero. However, as experimentally demonstrated in our work, we are able to observe these motional instabilities arising from purely static perturbations and wherein the frequencies satisfy equation (7).

5 Conclusion

We have carried out experiments to measure the motional instabilities of electrons in a Penning trap. These instabilities are caused by static perturbations of the potential in the trap and result in the electrons leaving the trap. They are found to occur when the characteristic frequencies obey a relation as in equation (7), a relation predicted theoretically, by standard methods of classical perturbation theory. Further, equation (15) shows that it is possible, for a given instability, to establish the lowest order of the perturbing potential that may be the cause of this instability.

These motional instabilities are of interest for investigating the nonlinear properties of ion traps. They can however, be the undesirable consequences of static and time dependent perturbations, as in mass spectrometry, or where storage of charged particles for relatively longer times is desired. It is also shown that a relation analogous to equation (7) holds good for those frequencies in storage rings at which the motion of the accelerated particles become unstable. Our work represents to the best of our knowledge, the first detailed experimental studies on these storage instabilities in Penning Trap in the presence of static perturbations.

Our experiments were supported by the Deutsche Forschungsgemeinschaft. C.A. thanks for support from the Alexander-von-Humboldt Stiftung. S.A. acknowledges support from the DST-DAAD Indo-German exchange Program. We acknowledge the help of Mr. S. Krause on some of the figures. Discussions with Professor M. Kretzschmar, with the theoretical analysis, are gratefully acknowledged.

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